Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2016

Mathematics

Paper 1

Higher Level

Friday 10th June Afternoon 2:00 – 4:30

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer **all nine** questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if you do not show all necessary work.

You may lose marks if you do not include appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here: [ ]

Section A — Concepts and Skills — 150 marks

Answer **all six** questions from this section.

Question 1 (25 marks)

(a) $(-4 + 3i)$ is one root of the equation $az^2 + bz + c = 0$, where $a, b, c \in \mathbb{R}$, and $i^2 = -1$. Write the other root.

(b) Use De Moivre's Theorem to express $(1 + i)^8$ in its simplest form.

(c) $(1 + i)$ is a root of the equation $z^2 + (-2 + i)z + 3 - i = 0$.
Find its other root in the form $m + ni$, where $m, n \in \mathbb{R}$, and $i^2 = -1$.

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Question 2 **(25 marks)**

(a) Find the range of values of x for which $|x - 4| \geq 2$, where $x \in \mathbb{R}$.

(b) Solve the simultaneous equations:

$$x^2 + xy + 2y^2 = 4$$
$$2x + 3y = -1.$$

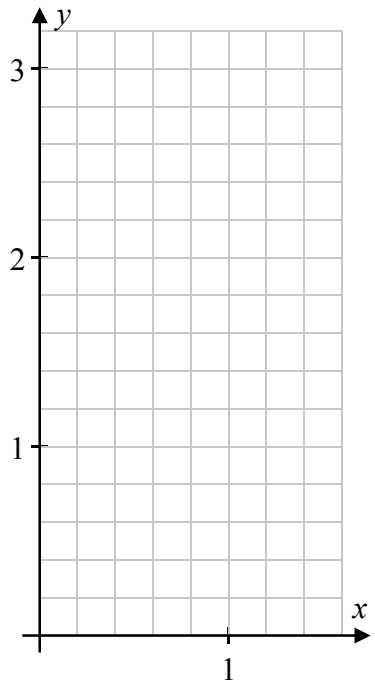
Question 3 **(25 marks)**

(a) **(i)** $f(x) = \frac{2}{e^x}$ and $g(x) = e^x - 1$, where $x \in \mathbb{R}$.

Complete the table below. Write your values correct to two decimal places where necessary.

x	0	0·5	1	ln(4)
$f(x) = \frac{2}{e^x}$				
$g(x) = e^x - 1$				

(ii) In the grid on the right, use the table to draw the graphs of $f(x)$ and $g(x)$ in the domain $0 \leq x \leq \ln(4)$. Label each graph clearly.



(iii) Use your graphs to estimate the value of x for which $f(x) = g(x)$.

(b) Solve $f(x) = g(x)$ using algebra.

Question 4 **(25 marks)**

(a) Prove by induction that $8^n - 1$ is divisible by 7 for all $n \in \mathbb{N}$.

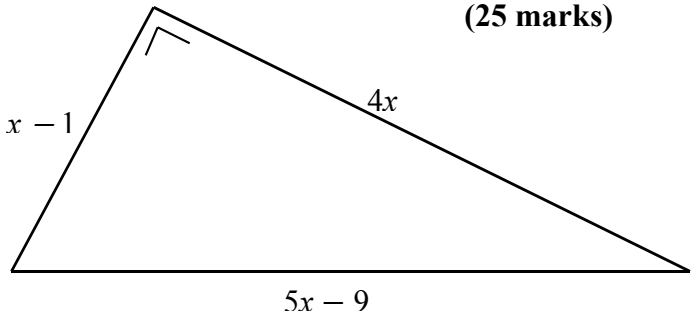
(b) Given $\log_a 2 = p$ and $\log_a 3 = q$, where $a > 0$, write each of the following in terms of p and q:

(i) $\log_a \frac{8}{3}$

(ii) $\log_a \frac{9a^2}{16}$.

Question 5 **(25 marks)**

(a) **(i)** The lengths of the sides of a right-angled triangle are given by the expressions $x - 1$, $4x$, and $5x - 9$, as shown in the diagram. Find the value of x.



(ii) Verify, with this value of x, that the lengths of the sides of the triangle above form a pythagorean triple.

(b) **(i)** Show that $f(x) = 3x - 2$, where $x \in \mathbb{R}$, is an injective function.

(ii) Given that $f(x) = 3x - 2$, where $x \in \mathbb{R}$, find a formula for f^{-1}, the inverse function of f. Show your work.

Question 6 **(25 marks)**

(a) Differentiate the function $(2x + 4)^2$ from first principles, with respect to x.

(b) **(i)** If $y = x\sin\left(\frac{1}{x}\right)$, find $\frac{dy}{dx}$.

(ii) Find the slope of the tangent to the curve $y = x\sin\left(\frac{1}{x}\right)$, when $x = \frac{4}{\pi}$.
Give your answer correct to two decimal places.

Section B — Contexts and Applications — 150 marks

Answer **all three** questions from this section.

Question 7 **(40 marks)**

(a) **(i)** Air is pumped into a spherical exercise ball at the rate of 250 cm^3 per second.
Find the rate at which the radius is increasing when the radius of the ball is 20 cm.
Give your answer in terms of π.

(ii) Find the rate at which the surface area of the ball is increasing when the radius of the ball is 20 cm.

(b) The inflated ball is kicked into the air from a point O on the ground. Taking O as the origin, $(x,\ f(x))$ approximately describes the path followed by the ball in the air, where

$$f(x) = -x^2 + 10x$$

and both x and $f(x)$ are measured in metres.

(i) Find the values of x when the ball is on the ground.

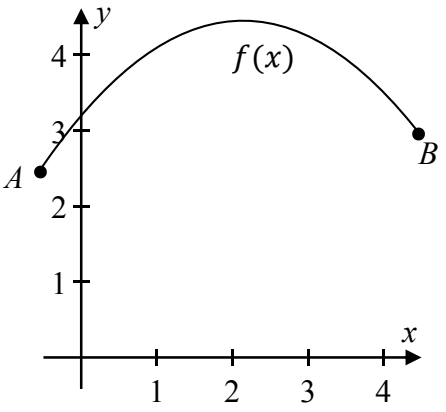
(ii) Find the average height of the ball above the ground, during the interval from when it is kicked until it hits the ground again.

Question 8 **(55 marks)**

(a) The diagram shows Sarah's first throw at the basket in a basketball game. The ball left her hands at A and entered the basket at B. Using the co-ordinate plane with $A(-0{\cdot}5,\ 2{\cdot}565)$ and $B(4{\cdot}5,\ 3{\cdot}05)$, the equation of the path of the centre of the ball is

$$f(x) = -0{\cdot}274x^2 + 1{\cdot}193x + 3{\cdot}23,$$

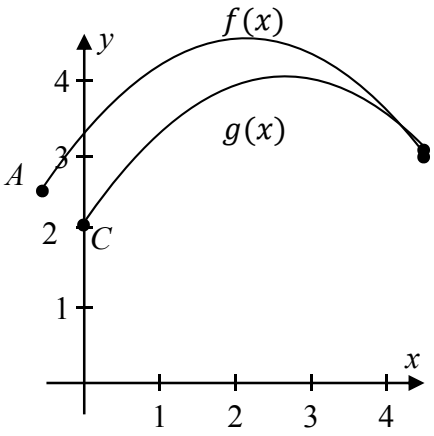
where both x and $f(x)$ are measured in metres.



(i) Find the maximum height reached by the centre of the ball, correct to three decimal places.

(ii) Find the acute angle to the horizontal at which the ball entered the basket. Give your answer correct to the nearest degree.

(iii) Sarah took a second throw. This throw followed the path of the parabola $g(x)$ as shown.
The ball left Sarah's hands at the point $C(0, 2)$.
The graph $y = g(x)$ is the image of the graph $y = f(x)$ under the translation which maps A onto C.
Using your result from part **a(i)**, show that the centre of this ball reached its maximum height at the point $(2{\cdot}677, 3{\cdot}964)$, correct to three decimal places.



(iv) Hence, or otherwise, find the equation of the parabola $g(x)$.

(b) The heptathlon is an Olympic competition. It consists of seven events including the 200 m race and the javelin. The scoring system uses formulas to calculate a score for each event. The table below shows the formulas for two of the events and the values of constants used in these formulas, where x is the time taken (in seconds) or distance achieved (in metres) by the competitor and y is the number of points scored in the event.

Event	***x***	**Formula**	***a***	***b***	***c***
200 m race	Time (s)	$y = a(b - x)^c$	4·99087	42·5	1·81
Javelin	Distance (m)	$y = a(x - b)^c$	15·9803	3·8	1·04

(i) In the heptathlon, Jessica ran 200 m in 23·8 s and threw the javelin 58·2 m. Use the formulas in the table to find the number of points she scored in each of these events, correct to the nearest point.

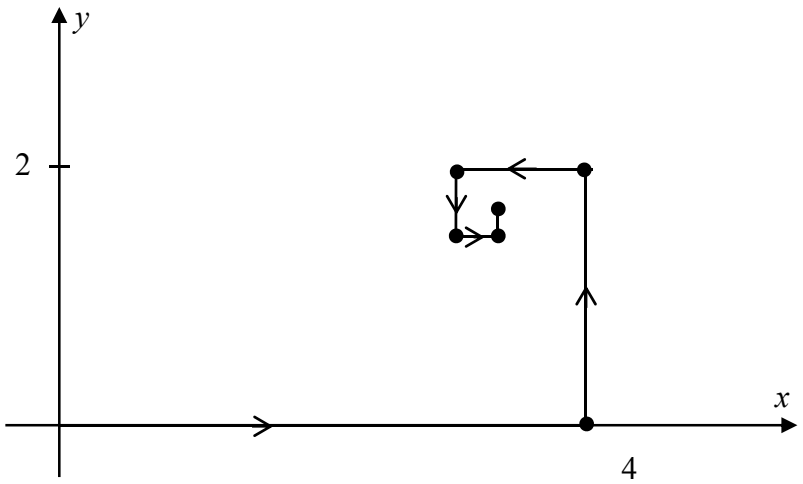
200 m | Javelin

(ii) The world record distance for the javelin, in the heptathlon, would merit a score of 1295 points. Find the world record distance for the javelin, in the heptathlon, correct to two decimal places.

(iii) The formula used to calculate the points for the 800 m race, in the heptathlon, is the same formula used for the 200 m race but with different constants.
Jessica ran the 800 m race in 2 minutes and 1·84 seconds which merited 1087 points.
If $a = 0{\cdot}11193$ and $b = 254$ for the 800 m race, find the value of c for this event, correct to two decimal places.

Question 9 **(55 marks)**

(a) At the first stage of a pattern, a point moves 4 units from the origin in the positive direction along the x-axis. For the second stage, it turns left and moves 2 units parallel to the y-axis. For the third stage, it turns left and moves 1 unit parallel to the x-axis.
At each stage, after the first one, the point turns left and moves half the distance of the previous stage, as shown.



(i) How many stages has the point completed when the total distance it has travelled, along its path, is 7·9375 units?

(ii) Find the maximum distance the point can move, along its path, if it continues in this pattern indefinitely.

(iii) Complete the second row of the table below showing the changes to the x co-ordinate, the first nine times the point moves to a new position. Hence, or otherwise, find the x co-ordinate and the y co-ordinate of the final position that the point is approaching, if it continues indefinitely in this pattern.

Stage	**1st**	**2nd**	**3rd**	**4th**	**5th**	**6th**	**7th**	**8th**	**9th**
Change in x	+4	0	–1						
Change in y									

(b) A male bee comes from an unfertilised egg, i.e. he has a female parent but he does not have a male parent. A female bee comes from a fertilised egg, i.e. she has a female parent and a male parent.

(i) The following diagram shows the ancestors of a certain male bee. We identify his generation as G_1 and our diagram goes back to G_4. Continue the diagram to G_5.

G_1	G_2	G_3	G_4	G_5
			Female	
		Female		
Male →	Female		Male	
		Male →	Female	

(ii) The number of ancestors of this bee in each generation can be calculated by the formula

$$G_{n+2} = G_{n+1} + G_n,$$

where $G_1 = 1$ and $G_2 = 1$, as in the diagram.
Use this formula to calculate the number of ancestors in G_6 and in G_7.

G_6 G_7

(iii) The number of ancestors in each generation can also be calculated by using the formula

$$G_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}.$$

Use this formula to verify the number of ancestors in G_3.

You may use this page for extra work.

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